Weak compressible magnetohydrodynamic turbulence in the solar corona

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This Letter presents a calculation of the power spectra of weakly turbulent Alfvén waves and fast magnetosonic waves ("fast waves") in low-β plasmas. It is shown that three-wave interactions transfer energy to high-frequency fast waves and, to a lesser extent, high-frequency Alfvén waves. MHD turbulence is thus a promising mechanism for producing the high-frequency waves needed to explain the anisotropic heating of minor ions in the solar corona.

PACS numbers: 52.35.Bj,52.35.Ra,95.30.Qd,96.60.Pb,96.60.Rd

The heating of the solar corona is a long-standing problem. Measurements taken with the Ultraviolet Coronagraph Spectrometer (UVCS) have provided important constraints on coronal heating, showing, for example, that $T_\perp \gg T_\parallel$ and $T_\perp \sim 10^8$ K for O^{+5} ions at a heliocentric distance r of roughly two solar radii, where $T_\perp$ and $T_\parallel$ are the temperatures for random motions perpendicular and parallel to the magnetic field $\boldsymbol{B}$. [1, 2] These measurements imply that the average magnetic moment $k_B T_\perp/B$ of O^{+5} ions increases rapidly with r and show that O^{+5} ions are heated by electric and magnetic-field fluctuations with frequencies ω comparable to or greater than the ions' cyclotron frequency Ω. (If $\omega \ll \Omega$, the average magnetic moment is almost exactly conserved.)

Different sources have been proposed for the required high-frequency waves, including reconnection events in the coronal base [3, 4, 5, 6], heat-flux-driven plasma instabilities [7], and magnetohydrodynamic (MHD) turbulence [8, 9, 10]. An apparent difficulty with this last source is the finding that in incompressible and weakly compressible MHD turbulence there is little or no cascade of energy to high frequencies. [11, 12, 13, 14, 15] However, incompressible and weakly compressible MHD neglect the fast magnetosonic wave ("fast wave"). In this Letter, a weak turbulence calculation is used to show that when fast waves are accounted for, MHD turbulence in low-β plasmas transfers energy to high-frequency fast waves and, to a lesser extent, high-frequency Alfvén waves. (In the corona, $\beta \equiv 8\pi p/B^2 \sim 0.01$, where p is the pressure.) The high-frequency waves produced by MHD turbulence may be of importance not only for coronal heating, but for particle acceleration in solar flares as well. [16]

The MHD momentum and induction equations with Laplacian viscosity and resistivity are

$$\rho\left(\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla v}\right) = -\boldsymbol{\nabla}\left(p + \frac{B^2}{8\pi}\right) + \frac{\boldsymbol{B}\cdot\boldsymbol{\nabla B}}{4\pi} + \rho\nu\nabla^2\boldsymbol{v} \quad (1)$$

and

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B}) + \eta\nabla^2\boldsymbol{B}, \quad (2)$$

where ρ is the density, $\boldsymbol{v}$ is the velocity, p is the pressure, and $\boldsymbol{B}$ is the magnetic field. In this Letter, the magnetic field is taken to consist of a uniform background field and a small-amplitude fluctuating field: $\boldsymbol{B} = B_0\hat{z} + \delta\boldsymbol{B}$. The pressure is discarded since β is taken to be $\ll 1$. The spatial Fourier transforms of $\boldsymbol{v}$ and $\boldsymbol{b} \equiv \delta\boldsymbol{B}/\sqrt{4\pi\rho}$ can be written

$$\boldsymbol{v}_k = v_{a,k}\hat{\boldsymbol{e}}_{a,k} + v_{f,k}\hat{\boldsymbol{k}}_\perp + v_{s,k}\hat{\boldsymbol{z}} \quad (3)$$

and

$$\boldsymbol{b}_k = b_{a,k}\hat{\boldsymbol{e}}_{a,k} + b_{f,k}\hat{\boldsymbol{e}}_{f,k}, \quad (4)$$

where $\hat{\boldsymbol{e}}_{a,k} = \hat{\boldsymbol{z}}\times\hat{\boldsymbol{k}}_\perp$ is the Alfvén-wave polarization vector at wave vector $\boldsymbol{k}$, $\hat{\boldsymbol{k}}_\perp = \boldsymbol{k}_\perp/k_\perp$, $\boldsymbol{k}_\perp = \boldsymbol{k} - k_z\hat{\boldsymbol{z}}$, and $\hat{\boldsymbol{e}}_{f,k} = \hat{\boldsymbol{e}}_{a,k}\times\boldsymbol{k}/k$. The Alfvén-wave amplitude is given by $a_k^\pm = v_{a,k} \pm b_{a,k}$, and (since $\beta \ll 1$) the fast-wave amplitude is given by $f_k^\pm = v_{f,k} \pm b_{f,k}$. Upon neglecting nonlinear terms in the momentum and induction equations, one obtains $\partial a_k^\pm/\partial t = \pm i k_z v_A a_k^\pm$ and $\partial f_k^\pm/\partial t = \pm i k v_A f_k^\pm$, where $v_A = B_0/\sqrt{4\pi\rho}$ is the Alfvén speed. Alfvén waves have frequency $\mp k_z v_A$ and propagate along magnetic field lines. Fast waves have frequency $\mp k v_A$ and can propagate in any direction. The $v_{s,k}\hat{\boldsymbol{z}}$ term in equation (3) corresponds to the slow magnetosonic wave, which has a frequency that approaches zero as $\beta \to 0$.

Weak turbulence consists of waves whose amplitudes are sufficiently small that nonlinear interactions between waves can be treated as a small perturbation to a wave's linear behavior. Weak turbulence theory is based on the assumptions of random wave phases and approximately Gaussian statistics. [17] These assumptions are problematic for acoustic turbulence, because sound waves propagating non-dispersively in the same direction interact coherently for long times. [17, 18] The same issue arises for fast waves, because of their acoustic-like dispersion relation. However, fast-wave interactions with Alfvén waves and slow magnetosonic waves limit the interaction time for pure fast-wave interactions, which may allow weak turbulence theory to apply to MHD at low β even if it does not apply to acoustic turbulence. Although this issue remains unresolved, weak turbulence theory is a valuable starting point for this difficult problem.

To simplify the analysis, the slow magnetosonic wave is neglected, the density ρ is taken to be a constant, and (to maintain energy conservation when ρ is held constant) the $\boldsymbol{v}\cdot\nabla\boldsymbol{v}$ term in equation (1) is replaced with $\boldsymbol{v}^A\cdot\nabla\boldsymbol{v}$, where $\boldsymbol{v}^A$ is the part of the velocity associated with Alfvén waves. A different approach was taken by [19], who included slow

waves but neglected three-wave interactions that did not involve slow waves. Further work including all the nonlinearities is needed. The Alfvén-wave and fast-wave power spectra for homogeneous turbulence are defined through the equations $\langle a_k^\pm (a_{k1}^\pm)^\star\rangle = A_k^\pm \delta(\boldsymbol{k}-\boldsymbol{k}_1)$, and $\langle f_k^\pm (f_{k1}^\pm)^\star\rangle = F_k^\pm \delta(\boldsymbol{k}-\boldsymbol{k}_1)$, where $\langle \dots \rangle$ denotes an ensemble average. It is assumed that $A_k^+ = A_k^- \equiv A_k$, that $F_k^+ = F_k^- \equiv F_k$, and that $\langle a_k^\pm f_{k1}^\pm\rangle = \langle a_k^\pm f_{k1}^\mp\rangle = 0$. Rotational symmetry about the z axis is also assumed, so that $A_k = A(k_\perp, k_z, t)$ and $F_k = F(k_\perp, k_z, t)$. Taking the small-ν and small-η limits and employing the standard weak-turbulence approximations, one obtains the wave kinetic equations,

$$
\begin{aligned}
\frac{\partial A_k}{\partial t} = \frac{\pi}{8v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\, \{ &\delta(q_z) 8 (p_\perp n \bar{l})^2 A_q (A_p - A_k) + \delta(k_z+p_z+q) M_1 [M_2 F_q (A_p - A_k) + M_3 A_p (F_q - A_k)] \\
&+ \delta(k_z + p_z - q) M_4 [M_5 F_q (A_p - A_k) + M_3 A_p (F_q - A_k)] + \delta(k_z + p - q)\, M_6 [M_7 F_q (F_p - A_k) + M_8 F_p (F_q - A_k)] \}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\frac{\partial F_k}{\partial t} = \frac{\pi}{8v_A}\int d^3p\, d^3q\, \delta(\boldsymbol{k}-\boldsymbol{p}-\boldsymbol{q})\, \{ &9\sin^2\theta [\delta(k-p-q) kq F_p (F_q - F_k) + \delta(k+p-q)(k^2 F_p F_q + kp F_q F_k - kq F_p F_k)] \\
&+ \delta(k - p_z + q_z) M_9 [M_{10} A_q (A_p - F_k) + M_{11} A_p (A_q - F_k)] + \delta(k - p_z - q) M_{12} [M_{13} F_q (A_p - F_k) + M_{14} A_p (F_q - F_k)] \\
&+ \delta(k + p_z - q) M_{15} [M_{16} F_q (A_p - F_k) + M_{17} A_p (F_q - F_k)] \} ,
\end{aligned} \tag{6}
$$

where

$$
\begin{aligned}
M_2 &= -p_\perp m - (\cos\alpha + 1/2)(k_\perp l + p_\perp m + q_\perp n), \\
M_3 &= 2k_\perp l + 2p_\perp m + q_\perp n, \\
M_5 &= -p_\perp m + (\cos\alpha - 1/2)(k_\perp l + p_\perp m + q_\perp n), \\
M_7 &= k_\perp \bar{l}(\cos\alpha - 1/2) + p_\perp \overline{m}/2 + \sin\alpha\, \overline{n}(2p - q/2), \\
M_8 &= k_\perp \bar{l}(\cos\psi + 1/2) - \sin\psi\, \overline{m}(2q - p/2) - q_\perp \overline{n}/2, \\
M_{10} &= p_\perp m + (\cos\theta + 1/2)(k_\perp l + p_\perp m + q_\perp n), \\
M_{11} &= q_\perp n + (-\cos\theta + 1/2)(k_\perp l + p_\perp m + q_\perp n), \\
M_{13} &= \sin\theta\, \bar{l}(-k + 2q) + p_\perp \overline{m}(\cos\theta - \cos\alpha) + \sin\alpha\, \overline{n}(q - 2k), \\
M_{14} &= \sin\theta\, \bar{l}(k/2 - 2q) + p_\perp \overline{m}(-\cos\theta + 1/2) - q_\perp \overline{n}/2, \\
M_{16} &= \sin\theta\, \bar{l}(-k + 2q) + p_\perp \overline{m}(\cos\alpha - \cos\theta) + \sin\alpha\, \overline{n}(q - 2k), \\
M_{17} &= \sin\theta\, \bar{l}(k/2 - 2q) + p_\perp \overline{m}(\cos\theta + 1/2) - q_\perp \overline{n}/2,
\end{aligned}
$$

$M_1 = M_2 + M_3$, $M_4 = M_5 + M_3$, $M_6 = M_7 + M_8$, $M_9 = M_{10} + M_{11}$, $M_{12} = M_{13} + M_{14}$, and $M_{15} = M_{16} + M_{17}$. The quantities θ, ψ, and α are the angles between $\hat{\boldsymbol{z}}$ and the wave vectors $\boldsymbol{k}$, $\boldsymbol{p}$, and $\boldsymbol{q}$, respectively. In the triangle with sides of lengths $k_\perp$, $p_\perp$, and $q_\perp$, the interior angles opposite the sides of length $k_\perp$, $p_\perp$, and $q_\perp$ are denoted σ_k, σ_p, and σ_q, and $l = \cos\sigma_k$, $m = \cos\sigma_p$, $n = \cos\sigma_q$, $\bar{l} = \sin\sigma_k$, $\overline{m} = \sin\sigma_p$, and $\overline{n} = \sin\sigma_q$. The above form of the wave kinetic equation makes use of the identities $k_\perp \cos(\sigma_q - \sigma_p) = q_\perp n + p_\perp m$ and $k_\perp \sin(\sigma_q - \sigma_p) = q_\perp \overline{n} - p_\perp \overline{m}$.

The right-hand sides of equations (5) and (6) (the "collision integrals") represent the effects of resonant three-wave interactions. The integrals sum over all possible wavenumber triads, while the delta functions restrict the sum to triads satisfying the resonance conditions $\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q}$ and $\omega_k = \omega_p + \omega_q$, where ω_k is the frequency at wavenumber $\boldsymbol{k}$. The equations $M_1 = M_2 + M_3$, $M_4 = M_5 + M_3$, etc imply that $\partial A_k/\partial t$ (or $\partial F_k/\partial t$) is positive at any wave number at which A_k (or F_k) vanishes, provided the spectra are positive at other wave numbers. The wave kinetic equations thus ensure that the spectra remain positive (realizability). Since the dissipative terms have not been included, equations (5) and (6) conserve the energy per unit mass $E = \int d^3k\,(A_k + F_k)/2$ and have an equipartition solution $F_k = A_k =$ constant.

The $\delta(q_z)$ in the collision integral of equation (5) is equivalent to $2v_A\delta(k_z v_A - p_z v_A + q_z v_A)$ and represents the frequency-matching condition for resonant interactions involving three Alfvén waves ("AAA interactions"). The part of the collision integral that contains this $\delta(q_z)$ is the same as the collision integral for AAA interactions in incompressible MHD. This term represents interactions between oppositely directed Alfvén waves, in which the field-line displacements caused by Alfvén waves travelling in one direction along the magnetic field [represented by $A(q_\perp, q_z = 0)$] distort Alfvén wave packets travelling in the opposite direction. At $k_z = 0$, only the AAA terms contribute to the right-hand side of equation (5), and the steady-state solution $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$ can be obtained analytically with the use of a Zakharov transformation, as in the incompressible case [14]. When $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$, and when non-AAA interactions are neglected, a Zakharov transformation yields $A_k \propto k_\perp^{-3}$ for any k_z. It can be seen from equation (5) that the time scale τ_A for AAA interactions to transfer Alfvén-wave energy from $k_\perp$ to $2k_\perp$ is determined by $A(k_\perp, k_z = 0)$ and is independent of k_z, consistent with physical descriptions of the Alfvén-wave cascade. [12, 13, 20] If the Alfvén-wave energy per unit mass $(\delta v_{\rm rms})^2$ is dominated by wavenumbers of order some characteristic wave number k_0, if the spectrum is quasi-isotropic at $k \sim k_0$, and if $A(k_\perp, k_z = 0) \propto k_\perp^{-3}$ for $k_\perp \gtrsim k_0$, then $\tau_A \simeq v_A/[k_\perp(\delta v_{\rm rms})^2]$ for $k_\perp \gg k_0$, as in the incompressible case. [12, 13]

The terms in the collision integral of equation (6) proportional to $\delta(k - p - q)$ and $\delta(k + p - q)$ represent resonant three-wave interactions involving only fast waves ("FFF interactions"). As can be seen from the delta functions, FFF interactions occur only when $\boldsymbol{k}$ is parallel or anti-parallel to

both $\boldsymbol{p}$ and $\boldsymbol{q}$, indicating that these interactions transfer energy radially in k-space. The FFF terms are the same as the collision integral for weak acoustic turbulence [18], up to an overall multiplicative factor proportional to $\sin^2\theta$, and represent a weak form of wave steepening. As $\sin\theta \to 0$, the acoustic-like FFF interactions become less efficient because the fast waves become less compressive. If the non-FFF terms are neglected, then a Zakharov transformation can be used to show that $F_k = c_1 g(\theta) k^{-7/2}$ is a steady-state solution to equation (6) for any function $g(\theta)$. When $F_k = c_1 g(\theta) k^{-7/2}$, the energy flux in FFF interactions per unit mass per unit solid angle in k-space, ε, can be obtained in the same way as for weak acoustic turbulence [18], and is given by $\varepsilon = 9\pi^2 c_1^2 \sin^2\theta g^2 c_2/16 v_A$, where $c_2 = \int_0^\infty dx \ln(1+x)[x(1+x)]^{-5/2}[(1+x)^{9/2} - x^{9/2} - 1] \simeq 26.2$. If the fast waves were forced isotropically and non-FFF interactions were ignored, then $g = 1/\sin\theta$ in steady state so that ε is independent of θ. The time scale τ_F for FFF interactions to transfer fast-wave energy from k to $2k$ can be estimated by dividing the fast-wave energy per unit solid angle between k and $2k$ by the energy flux ε. Ignoring numerical coefficients, one obtains $\tau_F \sim v_A/[c_1 \sin^2\theta g(\theta) k^{1/2}]$ for $F_k = c_1 g(\theta) k^{-7/2}$. If the fast-wave energy were dominated by wavenumbers of order some characteristic wave number k_0, with $F_k = c_1 g(\theta) k^{-7/2}$ for $k \gtrsim k_0$, then $c_1 \sim (\delta v_{\mathrm{rms},F})^2 k_0^{1/2}$, where $(\delta v_{\mathrm{rms},F})^2$ is the energy per unit mass in fast waves. In this case, $\tau_F \sim v_A/[(\delta v_{\mathrm{rms},F})^2 (k_0 k)^{1/2} \sin^2\theta g(\theta)]$ for $k \gg k_0$.

The terms in equations (5) and (6) containing M_1 through M_{17} correspond to three-wave interactions involving both Alfvén waves and fast waves ("AAF and AFF interactions"). Such interactions exchange energy between fast waves and Alfvén waves within resonant wavenumber triads. At small θ, the frequencies of fast waves and Alfvén waves are comparable, and AAF and AFF interactions are efficient. For example, if $F_k = c_1 k^{-7/2} \sin^{-1}\theta$ and $A_k \ll F_k$ at small θ, then when $\theta \ll 1$ the largest contribution to $\partial F_k/\partial t$ comes from the term proportional to $M_{13} F_q F_k$ and is $-F_k/\tau_{AF}$, where $\tau_{AF} = (15 v_A \sin\theta)/(23\pi^2 c_1 k^{1/2})$ to lowest order in θ, a time scale that is $\ll \tau_F$. The energy lost by fast waves in this case is transferred primarily to Alfvén waves at the same wavenumber through the term in equation (5) containing $M_8 F_p F_q$. If A_k grows until $A_k = F_k$ at small θ, then the term containing $M_{13} F_q F_k$ is cancelled by the term containing $M_{13} A_p F_k$ to lowest order in θ, largely stemming the loss of fast-wave energy. AAF and AFF interactions thus act to make $A_k \simeq F_k$ at small θ. However, the constant-energy-flux solution $A_k \simeq F_k \propto k^{-7/2}$ is unsustainable, because as k increases energy is lost from the small-θ part of k-space to high $k_\perp$ through AAA interactions faster than it is replenished from small k by FFF interactions ($\tau_F \propto k^{-1/2}$, $\tau_A \propto k^{-1}$). The energy flux in FFF interactions at small θ must thus decrease with k as fast-wave energy is drained into Alfvén waves and then transferred out to large $k_\perp$. This process causes F_k to steepen relative to $k^{-7/2}$ at small θ, and results in Alfvén-wave energy at $|k_z| \gg k_0$. On the other hand, for $\theta \gtrsim 45^\circ$, the frequencies of Alfvén waves and fast waves differ considerably, and AAF and AFF interactions are unable to make $A_k \simeq F_k$ at $k \gg k_0$. In this part of k-space, AAA and FFF interactions dominate the right-hand sides of equations (5) and (6), so that $F_k \propto k^{-7/2}$ and $A_k = h(k_z) k_\perp^{-3}$ within the inertial range, where $h(k_z)$ is some (decreasing) function of $|k_z|$.

To obtain quantitative solutions for A_k and F_k, equations (5) and (6) are integrated forward in time numerically with initial spectra $A_k = F_k = k^2 \exp(-k^2/k_0^2)$. The isotropic forcing term $c_3 k^2 \exp(-k^2/k_0^2)$ is added to the right-hand sides of both equations (5) and (6). The dissipation terms $-c_4 k^2 A_k$ and $-c_4 k^2 F_k$ are added to the right-hand sides of equations (5) and (6) respectively, with c_4 chosen so that in steady state dissipation truncates the spectra at a wavenumber that is $\gg k_0$. The numerical method conserves energy to machine accuracy in the absence of dissipation and forcing and will be described in a future publication. Steady-state spectra at late times are plotted in figures 1 and 2 and are consistent with the qualitative picture described above. The Alfvén-wave spectra are $\propto h(k_z) k_\perp^{-3}$ for $k_\perp \gg |k_z|$ within the inertial range. Although $h(k_z)$ decreases with increasing $|k_z|$, there is some Alfvén-wave energy at $|k_z| \gg k_0$, in contrast to the case of weak incompressible MHD turbulence. [14] At $\theta = 45^\circ$, F_k is $\propto k^{-7/2}$ and A_k drops off more steeply than $k^{-7/2}$. At $\theta = 7.1^\circ$, F_k falls off more rapidly than $k^{-7/2}$ and AAF and AFF interactions keep $A_k \simeq F_k$. The cascade of fast-wave energy to high frequencies was found previously by [21] in direct numerical simulations with strongly turbulent Alfvén waves and slow waves. In contrast to this Letter, these authors found an isotropic $k^{-7/2}$ fast-wave spectrum.

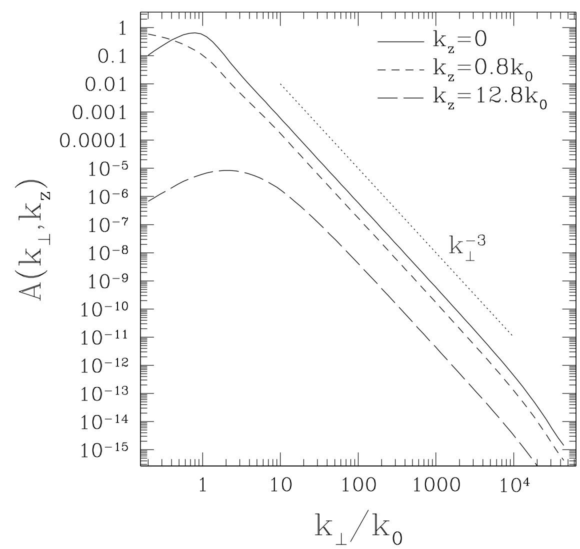

FIG. 1: Alfvén-wave power spectrum as a function of $k_\perp$ at different k_z.

The phenomenology described above can be applied more generally. For example, if the z-component of the phase velocity, $v_{\mathrm{ph},z}$, were initially positive for all the excited waves, and

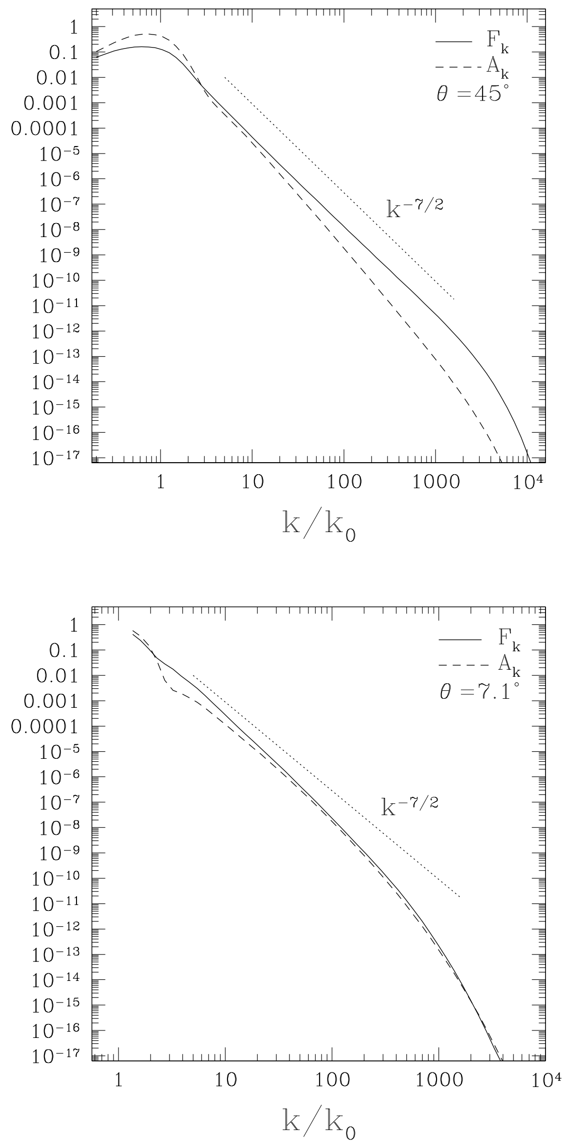

FIG. 2: *Top panel:* power spectra as a function of k at $\theta = 45°$. *Bottom panel:* power spectra as a function of k at $\theta = 7.1°$.

if there were no mechanism for generating Alfvén-waves with $k_z = 0$ and $v_{\rm ph,z} < 0$, then there would be no AAA interactions. In this case, FFF interactions would still transfer fast-wave energy to high frequencies, and AAF and AFF interactions would still cause A_k and F_k to become approximately equal at small θ, but the Alfvén-wave energy would not be swept out to large $k_\perp$ by AAA interactions. For waves with $v_{\rm ph,z} > 0$, one would thus expect F_k to obtain a constant-energy-flux $k^{-7/2}$ scaling for all θ with $A_k \simeq F_k$ at small θ. As a second example, if the initial excitation were primarily in Alfvén waves, as may be the case in the corona [3], and if A_k were quasi-isotropic at $k \sim k_0$, then AAF and AFF interactions would generate significant fast-wave energy at $k \sim k_0$, and FFF interactions would subsequently transfer fast-wave energy to higher frequencies. As a final example, if $\delta v_{\rm rms} \ll v_A$ but the Alfvén waves at small $|k_z|$ became strongly turbulent at $k_\perp$ larger than some transition wave number $k_{\rm tr}$, as in [12], then collisions between oppositely directed Alfvén wave packets would still transfer Alfvén wave energy at any k_z to larger $k_\perp$, but the cascade time for this process at $k_\perp > k_{\rm tr}$ would change from τ_A to a new value, $\tau_{A,\rm str} \propto k_\perp^{-2/3}$ [12]. The Alfvén waves in most of k-space and the fast waves would still be weakly turbulent because the linear periods of these waves would still be much shorter than the nonlinear time scales, and much of the weak-turbulence picture would still apply. In particular, F_k would be $\propto k^{-7/2}$ for $\theta \gtrsim 45°$, and A_k and F_k would be steeper than $k^{-7/2}$ at small θ and large k since τ_A, $\tau_{A,\rm str}$, and τ_{AF} are $\ll \tau_F$ in that part of k-space.

I thank Sebastian Galtier, Ellen Zweibel, Amitava Bhattacharjee, Marty Lee, and Joe Hollweg for helpful discussions. This work was supported by NASA under grant NNG 05GH39G and by NSF under grant AST 05-49577.

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